An Accelerated Monte Carlo Method for Lattice Quantum Gravity Simulations

Aaron Trowbridge

Physics Department
Syracuse University

Undergraduate Research Festival, April 30, 2021

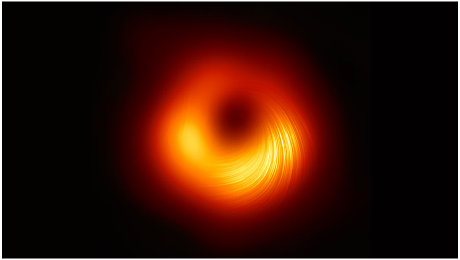
Table of Contents

Table of Contents

Unification

Gravity and Quantum Mechanics have to be considered simultaneously in certain scenarios:

- Black holes
- Dark Matter
- Cosmological Inflation



Physics progresses when theories are unified.

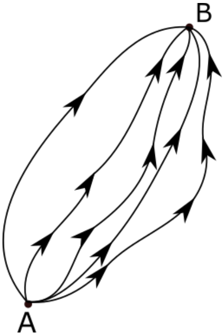
- Electromagnetism
- General Relativity
- Quantum Field Theory

Formulating Quantum Gravity

To formulate a quantum theory of gravity, we need to connect quantum field theory and general relativity, which can be accomplished via the path integral formalism developed by Richard Feynman.

The Feynman Path Integral

$$\langle x_f, t_f | x_i, t_i \rangle = \int \mathcal{D}[x(t)]\, e^{\frac{i}{\hbar}\int_{t_i}^{t_f} dt \mathcal{L}(x,\dot{x},t)}$$



The Gravitational Path Integral

In the continuum:

$$Z = \int \mathcal{D}[g]\, e^{-S_{EH}[g]}$$

with the Einstein-Hilbert action:

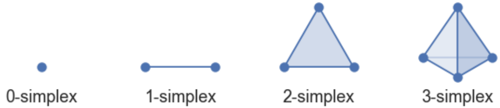
$$S_{EH} = -\frac{1}{16\pi G} \int d^4x \sqrt{g}(R - 2\Lambda)$$

On the lattice:

$$Z = \sum_T \frac{1}{C_T} \left[\prod_{j=1}^{N_2} \mathcal{O}(t_j)^\beta \right] e^{-S_{ER}}$$

with the Einstein-Regge action:

$$S_{ER} = -\kappa_2 N_2 + \kappa_4 N_4$$



Simplex Lattices

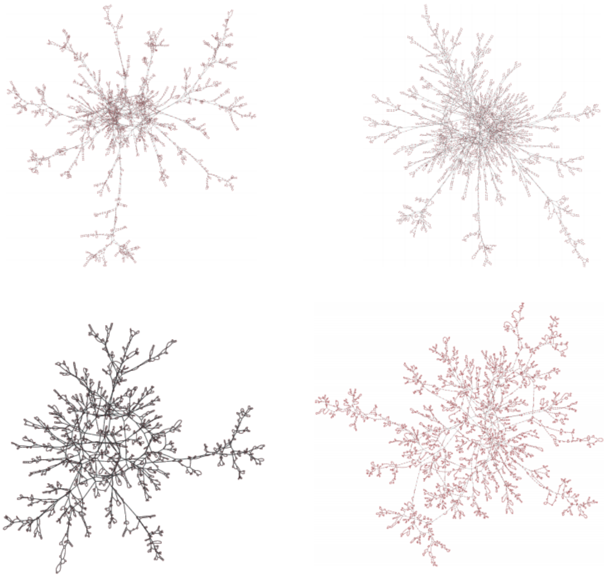


Table of Contents

The Metropolis-Hastings Algorithm

The goal of this method is to simulate a Markov process that, at equilibrium, satisfies a condition called detailed balance:

$$\pi_{AB} P_A = \pi_{BA} P_B$$

The probability for the system to be in state i is given by

$$P_i \propto e^{-\beta E_i} = e^{-S_i},$$

So,

$$\frac{\pi_{AB}}{\pi_{BA}} = \frac{P_B}{P_A} = e^{-(S_B - S_A)} = e^{-\Delta S} = e^{-\beta \Delta E}$$

is satisfied if

$$\pi_{AB} = \min\{1, e^{-\Delta S}\}$$

The 2-D Ising Model

On a 2-dimensional lattice of spin particles, the energy E of a configuration σ is given by

$$E(\sigma) = -\sum_{\langle i\, j \rangle} \sigma_i \sigma_j,$$

and the partition function is

$$Z = \sum_{\sigma} e^{-\beta E(\sigma)},$$

where $\beta = 1/kT$, and T is the temperature of the lattice.

Table of Contents

The Freeman Algorithm: A Rejection Free Method

As the acceptance probability goes down, rejecting moves gets computationally expensive. A new approach elminates rejections entirely:

1. Compute the probability p_i for every possible move and store in a list $p = (p_1, ..., p_N)$, and then compute a list $P = (P_1, ..., P_N)$, where $P_n = \sum_{i=1}^{n} p_i$
2. Generate a uniformly distributed random number $0 < r < P_N$, and make the i-th move if $P_{i-1} < r \leq P_i$
3. Update p and P with the updated transition probabilities for the new configuration
4. Record $n_{reject} = \left\lfloor \log_{1-\langle p \rangle}(r) \right\rfloor$, with $\langle p \rangle$ the average probability.

Binary Trees & Ponderance

Perk The Freeman Algorithm can be efficiently implemented using binary tree structures.

Problem The lattice gravity partition function has a global, geometry-dependent factor, so updating "capped" probabilities in a tree is not feasible.

Solution The detailed balance condition is also satisfied if $\pi_{AB} = \sqrt{e^{-\Delta S}} = e^{-\frac{\Delta S}{2}}$, which are "uncapped" probabilities we have called *ponderances*.

Testing on the Ising Model